Hypersonic Turbulent Boundary Layer with Pressure Gradients and Cross Flow

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Theme

AN experimental and theoretical investigation of the hypersonic compressible turbulent bounary-layer undergoing both normal and longitudinal adverse pressure gradients and cross flow is presented. The investigation is confined to the plane of symmetry on a compression flare in order to simulate the conditions existing at the centerline on an inlet of a jet engine. The cross flow is induced by connecting two lateral curved plates symmetrically on the sides of the flare. The integral equations of the compressible turbulent boundary-layer, which include terms referring to the cross flow, are solved numerically.

Content

The experiments were performed in a Mach 6 blowdown axisymmetric wind-tunnel, with a stagnation temperature of 800°R, a stagnation pressure of 1900 psia, and a freestream Reynolds number of 4.4×10^7 per foot. The model consisted of a streamlined centerbody followed by a compression flare 6.3 in. in length. The maximum turning angle of the flare was 43°. The over-all axial length of the centerbody, from the nozzle throat to the test section, was 77.3 in. The compression flare was equipped with pressure taps to measure the longitudinal and the peripheral pressure distributions, and thermocouples were used to measure the heat-transfer rates at the wall. The static pressure, total pressure, and total temperature profiles in the direction normal to the compression flare were measured with traversing probes at nine locations along the flare. Subsequently, the compression flare was cut laterally to connect two lateral curved plates symmetrically placed on each side of the flare to create an expansion in the peripheral direction and establish a favorable cross flow (C.F.), in addition to the normal and longitudinal pressure gradients.

Because of the large variation in the turning angle of the flare, the wall pressure increases from the freestream static pressure to a value approximately fifty times greater. The tests show no separation of the naturally established thick turbulent compressible boundary-layer along the centerbody. With the same initial conditions at the beginning of the compression flare, an inviscid method of characteristics solution shows that the compression waves coalesce into a shock outside the boundary-layer at the terminal part of the compression flare.

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The static pressure profiles show that the normal pressure gradient increases along the flare. The difference for the two cases, with and without cross flow, is about 10%.

The total pressure profiles show that the total pressures in the presence of cross flow are smaller than those without cross flow. However, there is no significant difference in the total temperature between the cases with and without cross flow.

The Mach number profiles and velocity profiles were reduced with the Rayleigh formula. The Mach number profiles indicate supersonic flow across the boundary-layer except for a very thin sublayer. The velocity profiles at the beginning of the flare coincide for both cases, with and without cross flow, but subsequently, there is a steepening of the velocity gradient in the case of cross flow, indicating a reduced adverse pressure gradient.

Plotting of the total temperature ratio vs velocity ratio shows that the total temperature ratio is higher than that which would be given by the linear Crocco relation at the beginning of the flare. Progressing longitudinally on the flare, the Walz quadratic relation is approached, and finally the values go back toward the linear Crocco relation.

The boundary-layer thickness at each station was determined by choosing the distance from the wall where the total pressure was approximately equal to the freestream stagnation pressure. The displacement, momentum, and energy thicknesses were integrated according to the measured profiles. These parameters are thinner for the case with cross flow and the suction effect of the cross flow is evident.

An existing theory[1,2] was extended in the present analysis. The momentum and energy integral equations were written in an orthogonal system of geodesic coordinates (x, φ, y). Assuming zero normal pressure gradient, the following governing equations were obtained,

$$\frac{d\theta}{dx} = -\left[(\bar{H}+2)\frac{1}{Ue}\frac{dUe}{dx} + \frac{1}{\rho_e}\frac{d\rho_e}{dx} + \frac{1}{r}\frac{dr}{dx}\right]\theta - \left(\frac{1}{r}\right)\left(\frac{\partial w}{\partial \varphi}\right)^*_{\text{Mer,}} \int_0^\delta \frac{\rho}{\rho_e}\left(1 - \frac{u}{Ue}\right)dy + \frac{C_f}{2} \tag{1}$$

$$\frac{d\Phi}{dx} = -\left[\frac{1}{U_e}\frac{dUe}{dx} + \frac{1}{\rho_e}\frac{d\rho_e}{dx} + \frac{1}{r}\frac{dr}{dx}\right]\Phi - \frac{1}{r}\left(\frac{dw}{\partial\varphi}\right)^*_{\text{Mer.}} \int_0^\delta\left(\frac{\rho}{\rho_e}\right)\left(1 - \frac{H^*}{H_e^*}\right)dy - \frac{q_w}{\rho_e U_e H_e^*} \tag{2}$$

where Φ is the energy thickness, and $H^* = H - H_w$.

$$\frac{\partial}{\partial x}\left(\frac{\partial w^*}{\partial \varphi}\right)_{\text{Mer,}} = -\frac{1}{r}\left(\frac{\partial w}{\partial \varphi}\right)^{*2}_{\text{Mer,}} - \frac{1}{r}\frac{dr}{dx}\left(\frac{\partial w}{\partial \varphi}\right)^*_{\text{Mer,}} - \frac{1}{r\gamma M_e^{\,2}}\left(\frac{1}{P_e}\frac{\partial^2 P_e}{\partial \varphi^2}\right)_{\text{Mer,}} \tag{3}$$

where w^* is the dimensionless velocity in the peripheral direction and the subscript Mer. denotes meridian.

The peripheral pressure distribution was approximately expressed by $P_e = A + B\cos\varphi + C\cos 2\varphi$. Hence,

$$\left(\frac{1}{P_e}\frac{\partial^2 P_e}{\partial \varphi^2}\right)_{\text{Mer,}} = \frac{1}{P_e}[-\text{B} - 4C] \tag{4}$$

where A, B, and C were determined by the experimental mea-

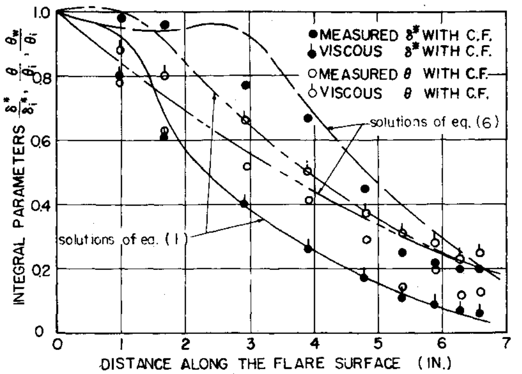

Fig. 1 Displacement and momentum thickness distributions.

surements of surface pressures at $\varphi = 0°$, $15°$, and $30°$ at each discrete location along the flare.

The skin friction related to a Reynolds number based on the momentum thickness was used.

$$C_f \equiv \frac{\tau_w}{\frac{1}{2}\rho_e U_e{}^2} = 0.246 e^{-1.56\bar{H}i}\left(\frac{U_e\theta}{\nu_{\text{ref}}}\right)^{-0.268}\left(\frac{T_e}{T_{\text{ref}}}\right)^{1.268} \tag{5}$$

The incompressible form factor $\bar{H}i$ was obtained from

$$\bar{H}_i = \left[\left(\bar{H} - \frac{\gamma-1}{2}M_e{}^2\right)\bigg/\left(1+\frac{\gamma-1}{2}M_e{}^2\right)\right] - \left(\frac{H_w}{H_e} - 1\right)\bar{H}_{i,F_P}$$

with

$$\bar{H}_{i,F_P} = 1.3$$

The reference properties were based on the reference temperature defined by

$$T_{\text{ref}}/T_e = 1 + 0.035\, M_e{}^2 + 0.45[(H_w/h_e) - 1]$$

A power law velocity profile, and the total enthaply profile, written in terms of the velocity profile and a constant as in Ref. 1, were assumed to be valid for the present analysis.

Reynolds analogy was used to correlate the heat-transfer rates and the skin friction. The momentum thickness based on the inviscid wall conditions was defined as

$$\rho_{wi} U_{w_i}^2 \theta_w = \int_0^\delta \rho u (U_{wi} - u)\, dy$$

where U_{wi} and ρ_{wi} were the inviscid velocity and density determined by the wall static pressure, and the wind-tunnel stagnation conditions. The momentum equation for θ_w was obtained,

$$\begin{aligned}\frac{d\theta_w}{dx} = &-\left[(\bar{H}+2)\frac{1}{U_{wi}}\frac{dU_{wi}}{dx} + \frac{1}{\rho_{wi}}\frac{d\rho_{wi}}{dx} + \frac{1}{r}\frac{dr}{dx}\right]\theta_w \\ &- \frac{1}{r}\left(\frac{\partial w}{\partial \varphi}\right)^*_{\text{Mer.}}\int_0^\delta\left(\frac{\rho}{\rho_{wi}}\right)\left(1-\frac{u}{U_{wi}}\right)dy + C_f/2 \\ &- \frac{1}{\rho_{wi}U_{wi}{}^2}\frac{d}{dx}\left[\int_0^\delta(\rho_w - p)\,dy\right] \\ &- \left(\frac{U_e - U_{wi}}{\rho_{wi}U_{wi}{}^2}\right)\left[\left(\frac{1}{r}\frac{dr}{dx}\right)\int_0^\delta \rho u\, dy + \frac{d}{dx}\left(\int_0^\delta \rho u\, dy\right)\right]\end{aligned} \tag{6}$$

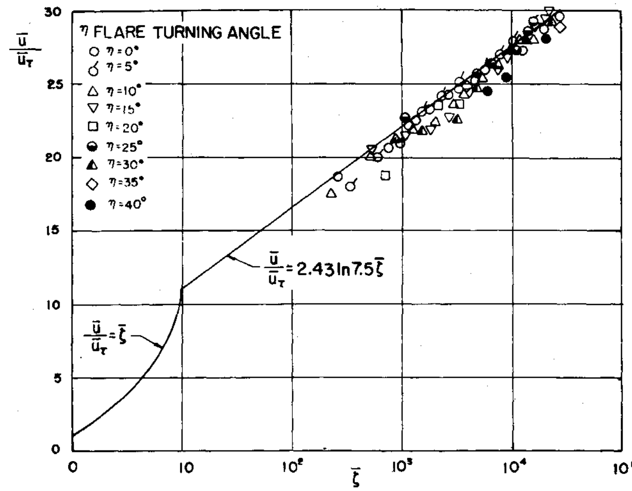

Fig. 2 Law of the wall correlation.

Utilizing the above equation, the effects of the longitudinal adverse pressure gradient, the normal pressure gradient, and the cross flow on the boundary-layer in the meridian plane were correlated.

Equations (1–6) were solved numerically with the initial conditions of $\theta_i = 0.0264$ in. and $\delta_i = 0.9$ in. at the beginning of the flare. By the Method of Ref. 3, the integral boundary-layer parameters due only to the viscous effect were calculated. The integral boundary-layer parameters were assumed to consist of two components, a viscous component and an inviscid component resulting from the normal pressure gradient. The measured parameters were obtained by integrating the experimental velocity and density profiles in the usual sense. The viscous parameters were obtained by subtracting the inviscid components from the measured quantities.

Theoretical and experimental results of displacement and momentum thicknesses for the case with cross flow are shown in Fig. 1. It can be seen that they are in good agreement. The measured heat-transfer rates are also well predicted by the theory. The method of Baronti and Libby, Ref. 4, was used to investigate the corresponding incompressible flow properties of the present work. As it can be seen in Fig. 2, the skin friction and velocity profiles satisfy the Law of the Wall correlation.

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